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AN INVESTIGATION OF THE PRESSURE STRESSES
IN A COMPACT, PLATE FIN, COUNTERFLOW
HEAT EXCHANGER

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AN INVESTIGATION OF THE PRESSURE STRESSES
IN A COMPACT, PLATE FIN, COUNTERFLOW HEAT EXCHANGER

by

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ABSTRACT

Regenerators have been proposed to increase thermal cycle efficiency of gas turbine power plants which are being considered for installation in certain classes of United States Navy ships. These regenerators are to preheat intake air and are currently designed as compact heat exchangers. The recommended heat exchanger configurations are either counterflow or crossflow. This analysis considers only the counterflow configuration.

Some of the structural problems associated with rectangular, compact heat exchanger design are discussed and certain structural design parameters are determined analytically. A digital computer program is presented for the analysis of the pressure effects in an idealized, two dimensional, rectangular heat exchanger. This program is written for a matrix analysis of an idealized heat exchanger with a cross section analogous to a multi-story building frame.

This investigation was conducted by Robert L. Corbett and David W. Stubbs during the period from December 1965 to May 1966, at the United States Naval Postgraduate School, Monterey, California.

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TABLE OF SYMBOLS

| | |
|----------------|---|
| a | Height of the heat exchanger module, in |
| b | Width of the heat exchanger module, in |
| c | Height of one flow channel in the heat exchanger, in |
| D | Plate flexural rigidity ($EI/1-\nu^2$), lb-in ² |
| E | Modulus of elasticity, lb/in ² |
| f | Distance between fins, in |
| H | Axial force, lb |
| I | Moment of inertia, in ⁴ |
| J ₁ | EI for the vertical side of the elemental frame, lb-in ² |
| J ₂ | EI for the horizontal side of the elemental frame, lb-in ² |
| K | Stiffness influence coefficient |
| L | Length of the heat exchanger module, in |
| M | Moment, in-lb |
| m | Frame member end moments, in-lb |
| p | Frame member end forces, lb |
| Q | Heat exchanger internal pressure, lb/in ² |
| t ₁ | Thickness of the vertical side of the elemental frame, in |
| t ₂ | Thickness of the horizontal side of the elemental frame, in |
| t _a | Thickness of the heat exchanger closure bars, in |
| t _b | Thickness of the heat exchanger top plate, in |
| t _f | Thickness of the heat exchanger fins, in |
| t _p | Thickness of the heat exchanger interior plates, in |
| V | Shear force, lb |
| ν | Poisson's ratio |
| θ | Frame joint rotations, radians |
| δ | Frame joint deflections, in |

1. INTRODUCTION

Several current projects in industry and one at the United States Naval Postgraduate School are actively concerned with the performance of gas turbine regenerators. The impetus for these investigations stems from a proposal to equip certain types of United States Navy ships with a power plant which will reduce, significantly, the delay time for transition from cruising speed to flank speed.

This desired flexibility in speed is planned to be achieved either by augmenting or replacing the current steam turbine installations with diesel or gas turbine power plants. In general, the efficiency of the diesel is superior to that of the pure gas turbine installations; but the advantage in weight and space for gas turbine engines is significant. These advantages, coupled with increased efficiency gained through the use of regenerators, make the gas turbine appealing for shipboard installations. Therefore the design, operation, and maintenance of gas turbine regenerators are important in the proposed plan for gas turbine shipboard installation.

Regenerator or heat exchanger requirements for different types of ships may differ in size, shape, and weight. A basic heat exchanger may be considered to consist of various combinations of small rectangular modules stacked or joined together in a manner that allows the complete heat exchanger to meet the heat transfer, geometry, and weight requirements for a particular installation. The rectangular shape is a logical choice for stacking and combining of modules and is simple to fabricate.

Present design requirements specify maximum pressures up to ten atmospheres, temperatures ranging from 500 to 1000 °F, and a service

life ranging from 20,000 to 60,000 hours. This latter requirement is severe, since structural integrity problems have occurred in similar heat exchangers after operating for much shorter periods of time.

The Bureau of Ships has suggested that the following areas be investigated:

1. Gas side fouling and air side salt precipitation
2. Thermal stress behavior
3. Thermal fatigue criteria
4. Materials evaluation
5. Header design and flow distribution
6. Fabrication and joining techniques
7. Advanced heat exchanger designs
8. Engine evaluation of advanced heat exchangers

This list indicates some of the areas which require investigation in order to define the requisite design criteria and to determine the order of reliability to be specified for a compact heat exchanger.

Preliminary stress investigations of typical rectangular heat exchangers have been accomplished to date. Few complete analyses of the complex state of stress within the heat exchanger core or supporting frame work have been made and those reported are, in general, sufficient only for the justification of the preliminary design. These result in heavier and more costly heat exchanger design, and possibly in reduced reliability. The authors believe that, through a careful analytical study to determine certain structural design parameters, heat exchanger design may be determined more readily.

The stresses in different parts of a heat exchanger are due primarily to the following: [2]

1. The difference in static and dynamic fluid pressures and atmospheric pressure.
2. The thermal expansion or contraction of component parts under:
 - a. Steady state conditions
 - b. Transient or thermal shock conditions.
3. Method of fabrication.

The values of these stresses are difficult to determine not only because of the complex structure but also since the value of pressure and temperature is a function of location and time.

The purpose of this investigation is to develop a basic method to study the effect of pressure on the design. Some of the problems relating to structural integrity are discussed and an attempt is made to isolate certain design parameters. The effect of some such parameters is qualitatively studied through the use of the high speed digital computer.

The investigation reported in this paper was conducted by the authors during the period from December 1965 to May 1966 at the United States Naval Postgraduate School, Monterey, California. Sincere appreciation is extended to the faculty and staff of the Aeronautical Engineering Department for their assistance. The authors are deeply indebted to Professor C. M. Smith for his invaluable encouragement and assistance throughout the project. The authors also wish to express their thanks to Professor C. H. Kahr for introducing them to and instructing them in structural analysis by matrix methods.

2. GENERAL

Heat exchangers of the cylindrical or tube type design have been analyzed and have proven to be structurally satisfactory. On the other hand, compact-rectangular heat exchangers have not been analyzed extensively for their structural action.

A number of analytical approaches to the structural problem were explored in this investigation of pressure effects. Some of the methods proved to be not entirely satisfactory and several of these are outlined in Appendix II.

The method of analysis adopted was limited to the core and was independent of the flange. The core is important from a heat transfer point of view, and is undoubtedly the most costly. The flange is not included in this investigation but its effect on structural design is sufficiently important to warrant further consideration.

In the analysis of the core, cross sections were considered to be similar to multi-story building frames with the closure bars forming main structural columns, the fins acting as partition walls or columns, and the interior plates acting as floors. Exploratory design parameters were determined by considering a rectangular frame without interior tie-bars or constraints. The effect of these parameters on the lattice-like frame composing the core cross section was then investigated.

3. ANALYSIS

For the purpose of this investigation, a rectangular core module for a counterflow heat exchanger was postulated. In this module the geometry and material properties were assumed to be constant over the length; the pressure was assumed to be constant in all channels. The cross section was divided into a pattern of small rectangular passages by the interior plates and fins. All end discontinuities resulting from attachment of the core to flanges were neglected. This idealized core module is presented in Figure 1.

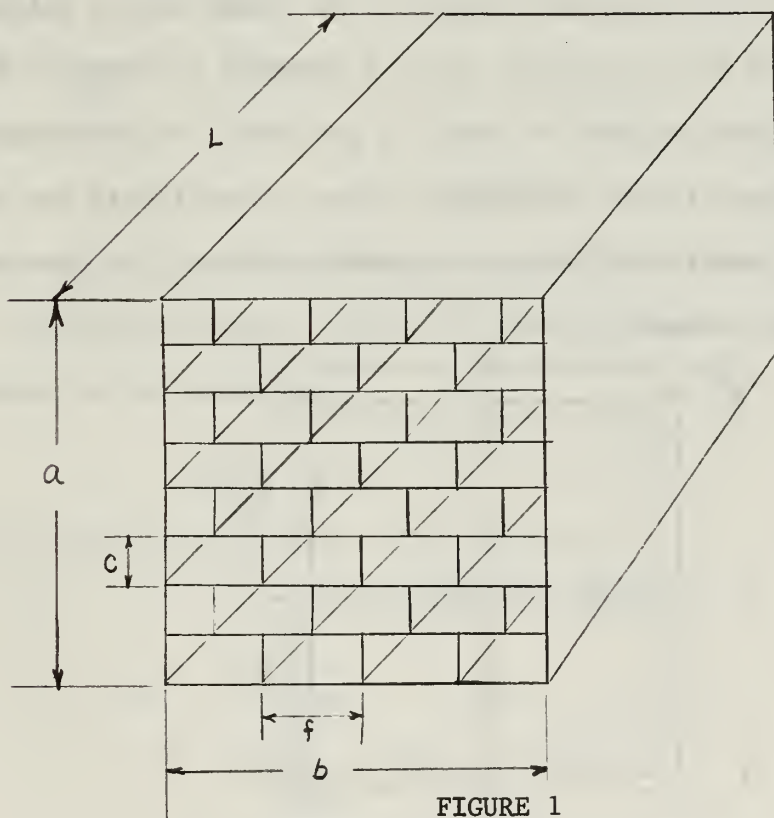


FIGURE 1

IDEALIZED COUNTERFLOW CORE MODULE

This module represents only a small part of the complete regenerator heat exchanger. Different configurations would have a number of these modules arranged in a manner suitable to the operational environment of the ship.

Close inspection of the idealized core module reveals that the fundamental element involved in the cross section is an open rectangular frame. Many of these basic frames are contained in a module cross section and two or more modules are stacked together to form a complete heat exchanger.

Since the fundamental element is an open rectangular frame, an analysis was performed on a simple frame in order to isolate parameters which may affect or define structural considerations for design. [1] In this analysis the frame was assumed to be loaded with a uniform internal pressure and to have two axes of symmetry as shown in Figure 2. While these assumptions are not strictly applicable to the arrangement of elemental frames in the core module, they do facilitate the analysis and provide information as to which parameters affect the stresses in each member of the frame.

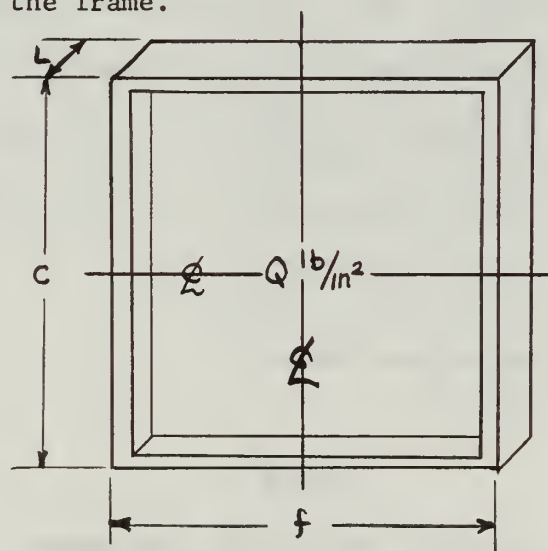


FIGURE 2
ELEMENTAL FRAME, TYPICAL

The two axes of symmetry which are shown in Figure 2 permit the analysis to be made for one quarter of the elemental frame; the required boundary conditions are defined through considerations of symmetry.

The frame representing one quarter of the elemental frame is shown in Figure 3.

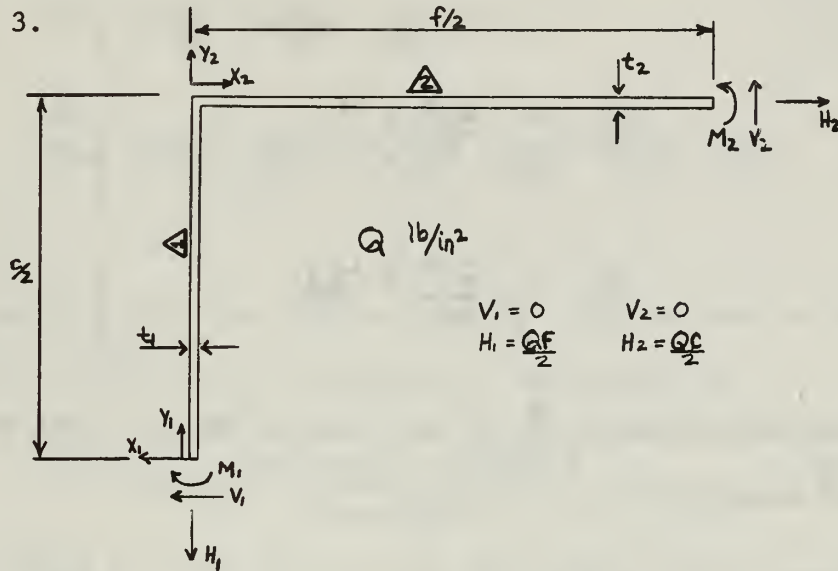


FIGURE 3

ONE QUARTER OF AN ELEMENTAL FRAME

Considering member $\triangle 1$ of the frame in Figure 3, the definitive differential equation is:

$$J_1 \frac{d^4 y_1}{dx_1^4} = Q \quad (1)$$

and the applicable boundary conditions are:

$$a. \quad J_1 \frac{d^3 y_1}{dx_1^3} \Big|_{x_1=0} = 0$$

$$b. \quad J_1 \frac{d^2 y_1}{dx_1^2} \Big|_{x_1=0} = M_1$$

$$c. \quad \frac{dy_1}{dx_1} \Big|_{x_1=c/2} = 0$$

$$d. \quad y_1 \Big|_{x_1=c/2} = 0$$

Successive integration of Equation 1, with the application of boundary conditions yields:

$$\left. \begin{aligned} V_{(x_1)} &= Qx_1 \\ M_{(x_1)} &= \frac{Qx_1^2}{2} + M_1 \\ \Theta_{(x_1)} &= \frac{Qx_1^3}{6J_1} + \frac{M_1x_1}{J_1} \end{aligned} \right] \quad (2)$$

Now considering member Δ of the frame in Figure 3, the definitive differential equation is:

$$J_2 \frac{d^4 y_2}{dx_2^4} = Q \quad (3)$$

and the appropriate boundary conditions are:

$$a. \quad J_2 \left. \frac{d^3 y_2}{dx_2^3} \right|_{x_2=0} = -H_1 = -\frac{Qf}{2}$$

$$b. \quad J_2 \left. \frac{d^2 y_2}{dx_2^2} \right|_{x_2=0} = \frac{Qc^2}{8} + M_1$$

$$c. \quad \left. \frac{dy_2}{dx_2} \right|_{x_2=0} = \frac{Qc^3}{48J_1} + \frac{M_1c}{2J_1}$$

$$d. \quad y_2 \Big|_{x_2=0} = 0$$

Integration of Equation 3 and the introduction of the above

boundary conditions leads to the following expressions:

$$\left. \begin{aligned} V_{(X_2)} &= QX_2 - \frac{Qf}{2} \\ M_{(X_2)} &= \frac{QX_2^2}{2} - \frac{QfX_2}{2} + \frac{Qc^2}{8} + M_1 \\ \Theta_{(X_2)} &= \frac{QX_2^3}{6J_2} - \frac{QfX_2^2}{4J_2} + \frac{Qc^2X_2}{8J_2} + \frac{Qc^3}{48J_1} + M_1\left(\frac{X_2}{J_2} + \frac{c^2}{2J_1}\right) \end{aligned} \right\} \quad (4)$$

From symmetry, the slope at the point $X_2 = f/2$ is equal to zero. This constraint allows the determination of the moment, M_1 .

$$M_1 = \frac{Q}{24} \left[\frac{2f^3 - 3c^2f - c^3(J_2/J_1)}{f + c(J_2/J_1)} \right] \quad (5)$$

Assuming that the sides of the frame do not differ greatly in length or in bending stiffness, the maximum moment occurs at the corners of the frame. From Equation 2, the maximum moment can be expressed as:

$$M_{MAX} = \frac{Qc^2}{24} \left\{ 3 + \left[\frac{2(f/c)^3 - 3(f/c) - (J_2/J_1)}{(f/c) + (J_2/J_1)} \right] \right\} \quad (6)$$

The maximum shear force on the horizontal side of the frame is:

$$V_{(X_2) MAX} = -\frac{Qf}{2}$$

and the maximum shear force on the vertical side is:

$$V_{(X_1) MAX} = -\frac{Qc}{2}$$

Equation 6 indicates that J_1 , J_2 , f/c , and Q are parameters which affect the choice of design from a structural point of view. Based on the assumption that the pressure and side dimensions will be dictated by requirements for flow and heat transfer, f/c and Q may be fixed for

a given design for reasons other than structural. Thus the structural designer may have little control over the magnitude of the shear forces. The magnitude of the moments may be controlled by varying J_1 and J_2 . Therefore the structural designer, by carefully choosing the appropriate values of member thickness, may be able to optimize the design for the idealized frame element.

In order to study the effect of varying member thicknesses, t_1 and t_2 , a two dimensional model of the heat exchanger module was investigated. A diagram of this model is shown in Figure 4. This model

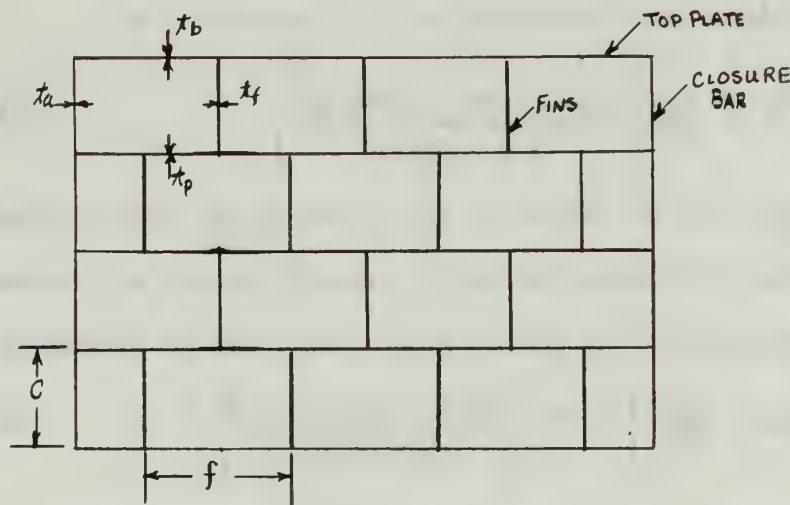


FIGURE 4

TWO DIMENSIONAL HEAT EXCHANGER MODEL

shows that t_b , t_a , t_p , and t_f are analogous to the t_1 and t_2 shown in Figure 3. The effect of varying t_a , t_b , t_p , and t_f within the model chosen is not obvious and cannot be defined in terms of expressions derived for moment and shear for the simple frame.

In two dimensionalizing the model, the top and bottom plates, the interior plates, and the closure bars are in reality three dimensional

plates which have been idealized as two dimensional beams by assuming an elemental cross section of the core. The Plate Flexural Rigidity, D , is used for each element to include this effect.

Slope and deflection methods of analysis are adaptable to matrix methods which in turn are easily programmed for solution by the high speed digital computer. The computer program, as used, is included in Appendix I with a set of answers for the chosen model. This program was prepared from considerations of the following.

The stiffness method of matrix solution is a form of the slope and deflection method as proposed by M. L. Pei. [4] The basic element for this method consists of a beam under the action of end moments m_1 and m_2 and end forces P_3 and P_4 as shown in Figure 5.

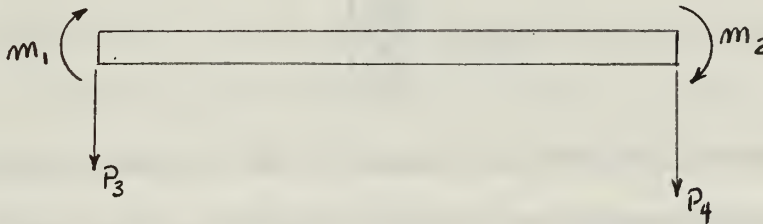


FIGURE 5

BASIC ELEMENT, SLOPE AND DEFLECTION ANALYSIS

Let the beam undergo end rotations θ_1 and θ_2 as well as end displacements δ_3 and δ_4 . The positive directions of these displacements are as defined in Figure 6.



FIGURE 6

BASIC ELEMENT DEFLECTIONS, SLOPE AND DEFLECTION ANALYSIS

This beam then, has four degrees of freedom represented by the displacement matrix:

$$\{\theta\} = \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \delta_3 \\ \delta_4 \end{Bmatrix}$$

Correspondingly, loads are represented by the load matrix:

$$\{m\} = \begin{Bmatrix} m_1 \\ m_2 \\ P_3 \\ P_4 \end{Bmatrix}$$

These loads and deflections are related by the stiffness influence coefficient K which depends upon the cross section and the modulus of elasticity of the beam. These equations may be expressed conveniently in matrix notation as follows:

$$\{m\} = [K] \{\theta\}$$

For the single member example given above, this general equation becomes:

$$m_1 = K_{11}\theta_1 + K_{12}\theta_2 + K_{13}\theta_3 + K_{14}\delta_4$$

$$m_2 = K_{21}\theta_1 + K_{22}\theta_2 + K_{23}\theta_3 + K_{24}\delta_4$$

$$P_3 = K_{31}\theta_1 + K_{32}\theta_2 + K_{33}\theta_3 + K_{34}\delta_4$$

$$P_4 = K_{41}\theta_1 + K_{42}\theta_2 + K_{43}\theta_3 + K_{44}\delta_4$$

Hence the "K" matrix is expressed as:

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix}$$

The loading $\{m\}$ for the structure is known, so the rotations and deflections of the member ends, or more generally, the joints in the structure, may be determined from:

$$\{\theta\} = [K]^{-1}\{m\}$$

The application of this method to a general two dimensional frame consists of computing the elemental "K" matrix for each element of the frame and then adding these elemental "K" matrices to form the total "K" matrix for the frame. The "K" matrix is then inverted, as indicated by the above equation, to determine the frame joint deflections.

Using the frame joint deflections the resulting element loads are found from:

$$\{m\} = [K]\{\theta\}$$

where $[K]$ is the elemental matrix and $\{\theta\}$ is the deflection of the joints in which the element terminates. These elemental loads are the

element internal loads. These internal loads describe the degree of relaxation from that of a fixed joint. The external loads or element reactions may then be determined from:

$$M_{FIXED} = M_{INTERNAL} + M_{REACTION}$$

These reactions are the desired results from which stresses may be calculated.

The process of calculating loads in this way is very laborious, particularly when it is necessary to invert a large "K" matrix. At the same time the process involved is purely a mechanical one which may be computerized.

4. RESULTS

The computer was used to determine the effect of varying t_a , t_b , and t_p using the following variations:

| <u>Basic Configuration</u> | <u>Variations</u> |
|----------------------------|-------------------|
| $t_a = .04$ in | .04 in -- .25 in |
| $t_b = .04$ in | .04 in -- .25 in |
| $t_p = .04$ in | .02 in -- .06 in |
| $t_f = .01$ in | Constant |
| $f = 0.75$ in | Constant |
| $c = 0.5$ in | Constant |

Maximum normal stresses and shear stresses for the variations shown were calculated. The shearing stress variations are functions of fin spacing and channel height, neither of which was varied since they were assumed to be dictated by thermodynamic considerations. The shearing stress variations were small when compared to the normal stress variations. For this reason the variations in shearing stresses were assumed to be negligible for the parameters being investigated. Graphs showing the effect of the changes on shearing stresses are presented in Figures 8 and 9.

Maximum normal stress variations for varying closure bar thickness are plotted in Figure 10. Maximum stresses are computed from:

Maximum normal stress - bending stress plus axial stress.

As the thickness of the closure bar decreases;

stress in the closure bar is increased rapidly,

stress in the top plate is decreased rapidly,

stresses in the interior plates and fins are decreased gradually.

Since the stresses in the closure bars increase rapidly, this method for reducing stresses in the model is considered marginal.

Maximum normal stress variations for varying interior plate thickness are shown in Figure 11. This graph indicates that for an increasing interior plate thickness, the stresses in all members decrease rapidly. However, increasing the thickness of the interior plates rapidly reduces the heat transfer effectiveness between channels. Since heat transfer is the main purpose of the heat exchanger, this method of reducing the stresses must be used judiciously.

Maximum normal stress variations for varying top plate thickness reduces the stresses in all members as long as the top plate is not too thick, see Figure 12. Beyond a certain top plate thickness the stresses in the top plate and the fins begin to increase. However, further thickness increase in the top plate is not necessarily detrimental as the closure bar stresses continue to fall off rapidly. At the point at which the top plate and the fin stresses reach a minimum and the closure bar stress reaches a local minimum, there is a significant shift in the location of maximum stress both in the closure bars and in the top plate. This maximum stress location shift results in a definite dip in the maximum stress values. Since variation of the thickness of the top plate has no basic undesirable effects, it is most advantageous to make initial stress reductions in the model by variation of top plate thickness.

For any given heat exchanger this computerized method of solution is useful in the determination of the range of pressure stresses in the module. A computer program and detailed instructions for its use are presented in Appendix I.

5. CONCLUSIONS AND RECOMMENDATIONS

The stresses due to internal pressure are a significant part of the stresses in a rectangular, compact heat exchanger. The value of these stresses for a pressure of 10 atmospheres may be on the order of sixty to ninety percent of design allowables.

The value of stresses in the lattice-like frame which composes the heat exchanger core can be controlled through variations in the thickness of the individual elements involved.

An optimum ratio of the sum of the stiffness of the top plate and interior plates to the stiffness of the closure bars is implied. This optimum should consider heat transfer as well as structural requirements.

Consideration should be given to increasing the thickness of the top plate and/or the thickness of the interior plates in order to achieve the required stiffness for these members. This approach can lead to an optimum stress level and distribution for an internal pressure loading.

The matrix analysis and the computer program used in this analysis should be expanded to provide solutions for three-dimensional problems with thermal as well as pressure loads.

The compatibility of the flange and core is an important structural consideration. This should be thoroughly investigated in conjunction with further analysis of the core.

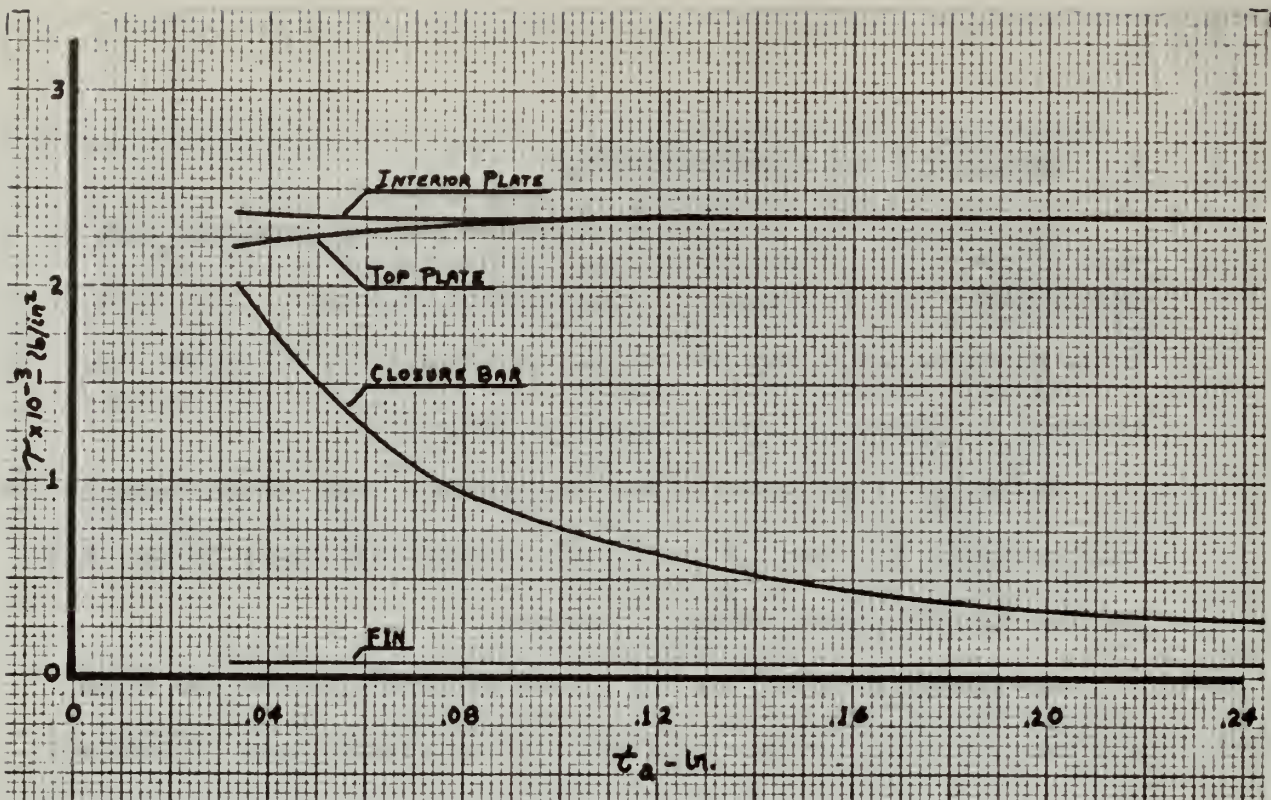


FIGURE 7. MAXIMUM SHEAR STRESS VERSUS CLOSURE BAR THICKNESS

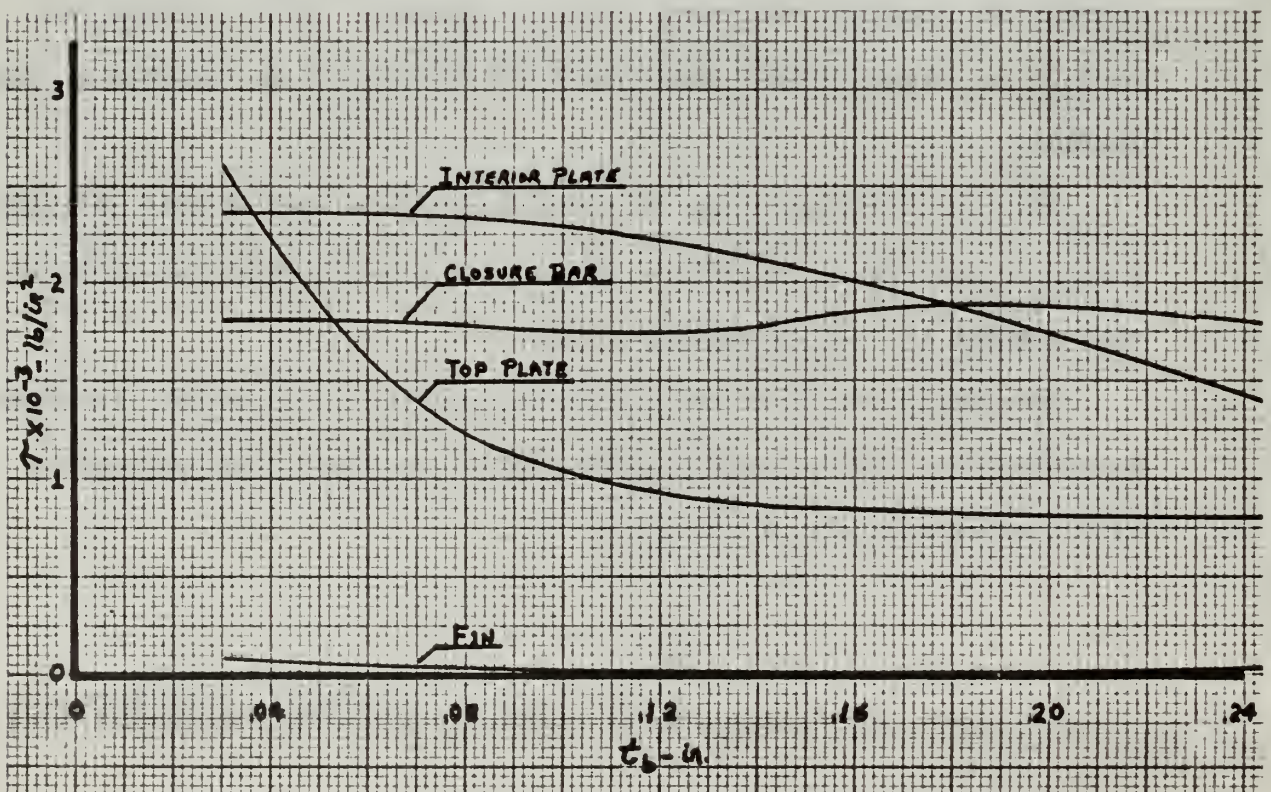


FIGURE 8. MAXIMUM SHEAR STRESS VERSUS TOP PLATE THICKNESS

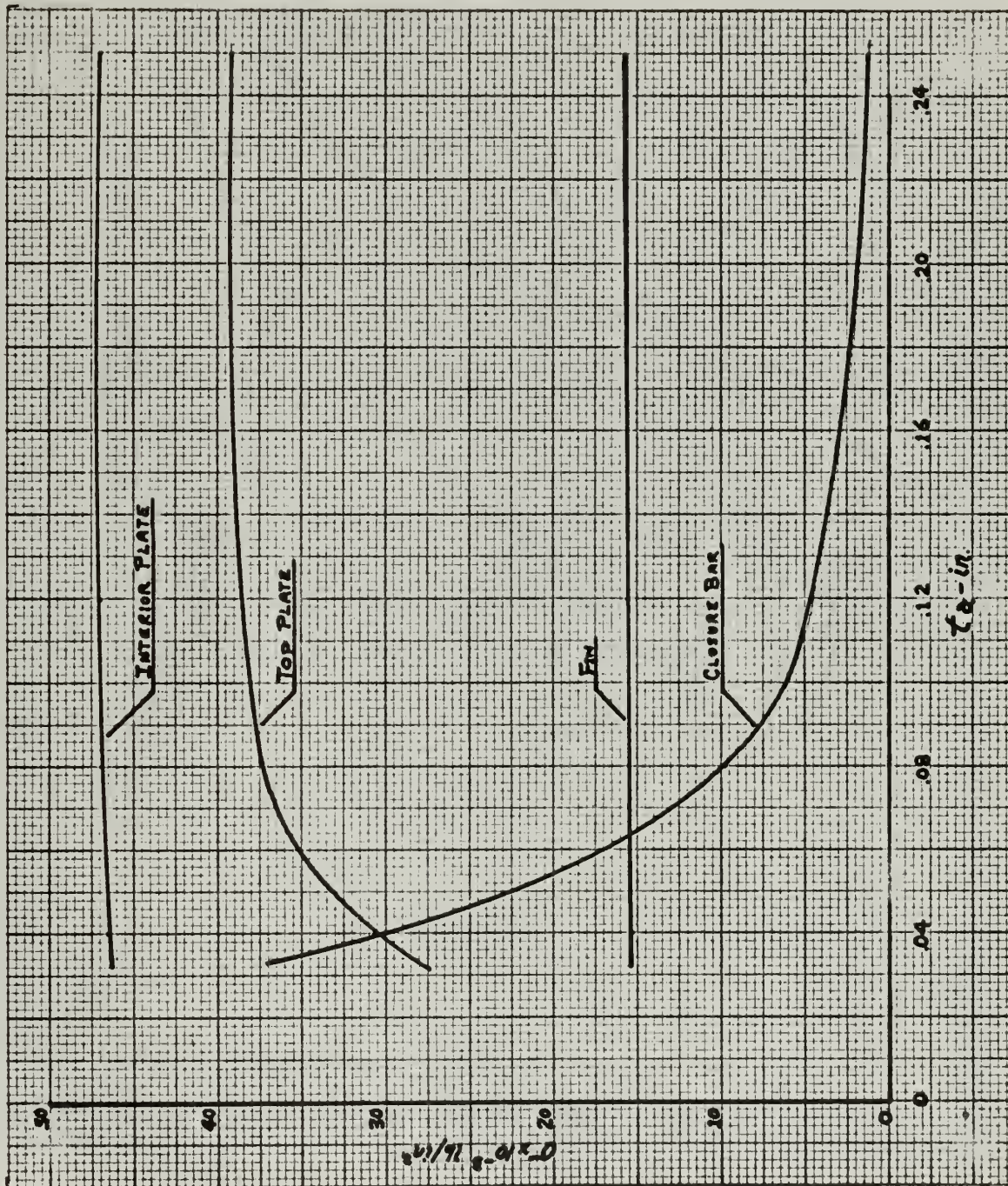


FIGURE 9. MAXIMUM NORMAL STRESS VERSUS CLOSURE BAR THICKNESS

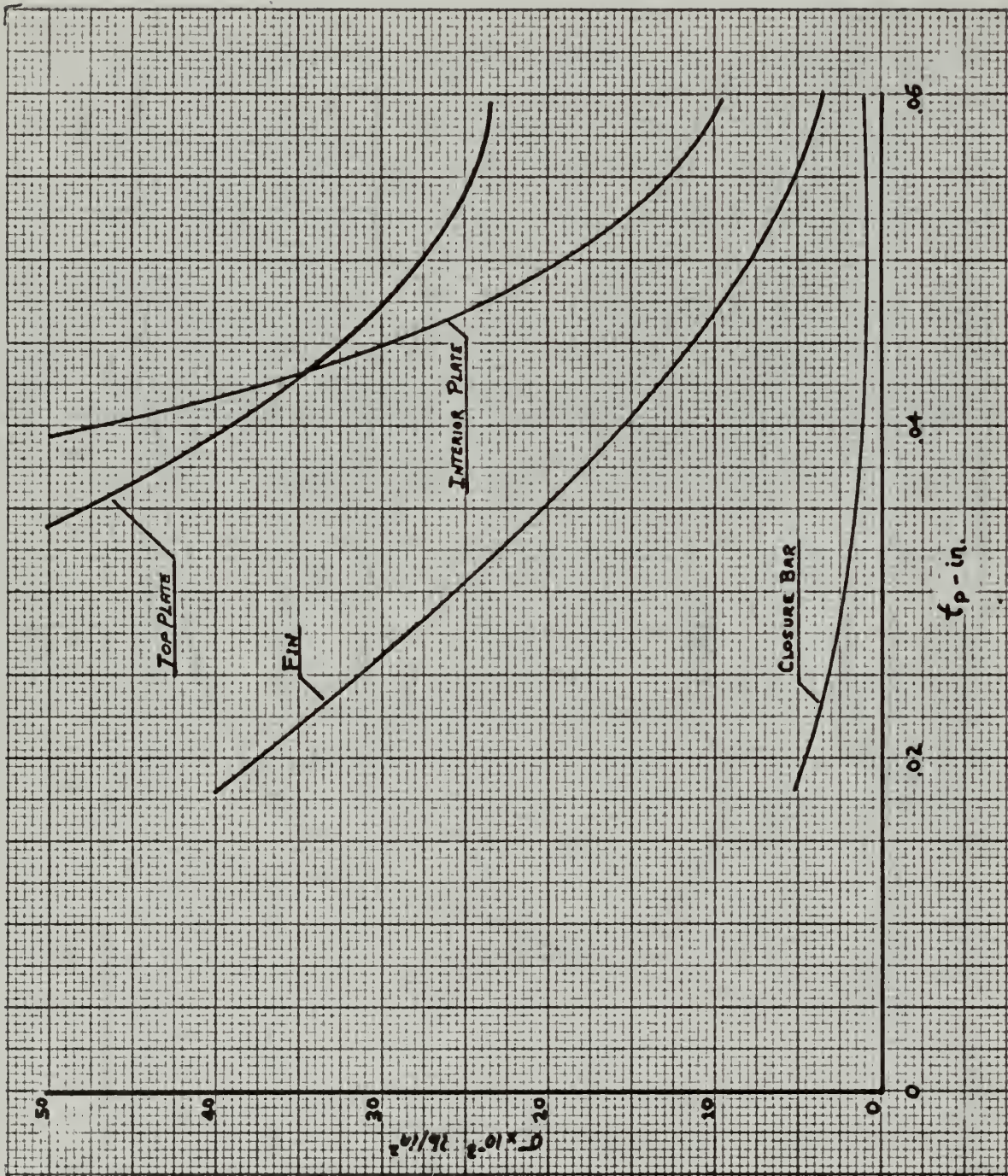


FIGURE 10. MAXIMUM NORMAL STRESS VERSUS INTERIOR PLATE THICKNESS

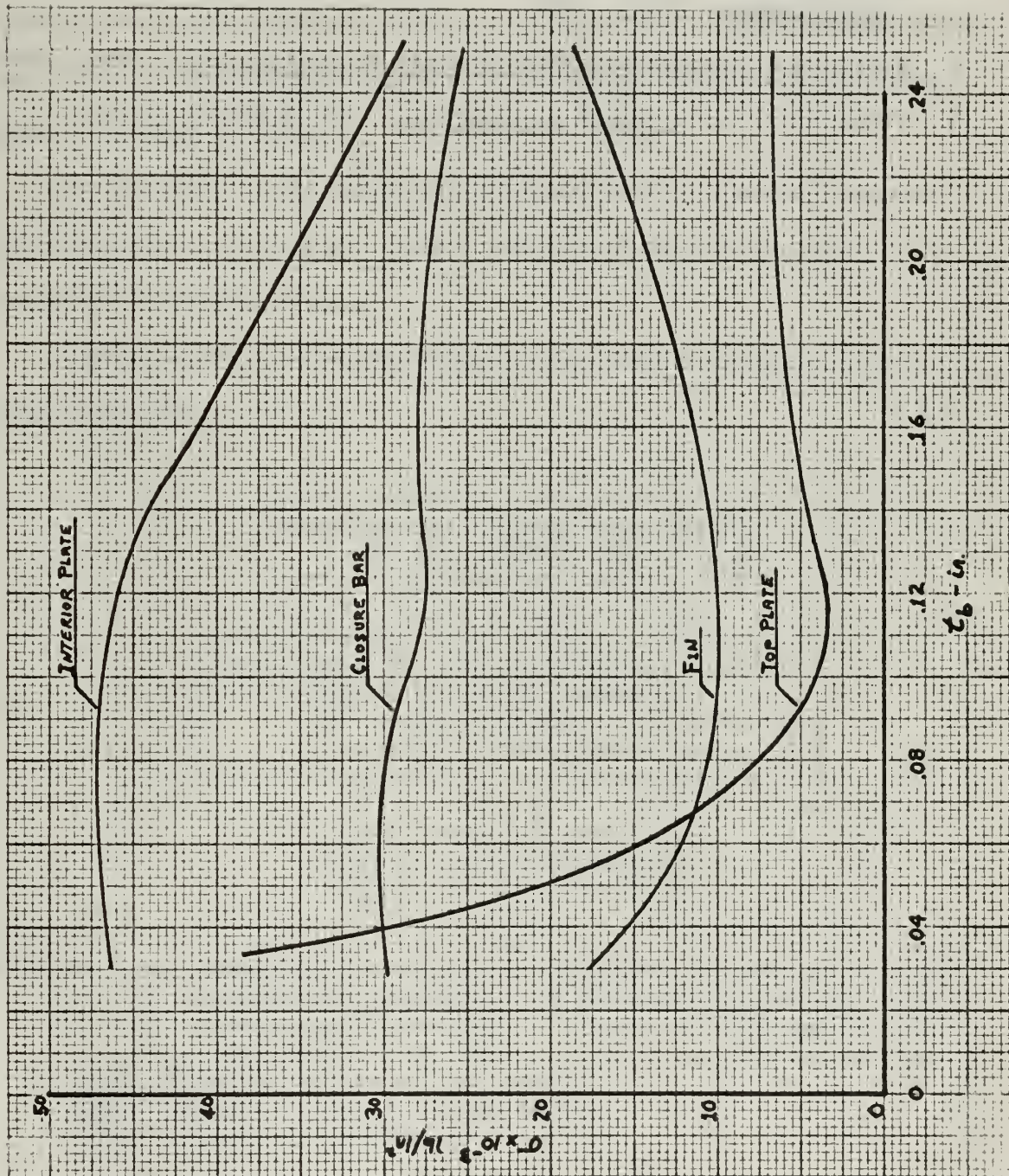


FIGURE 11. MAXIMUM NORMAL STRESS VERSUS TOP PLATE THICKNESS

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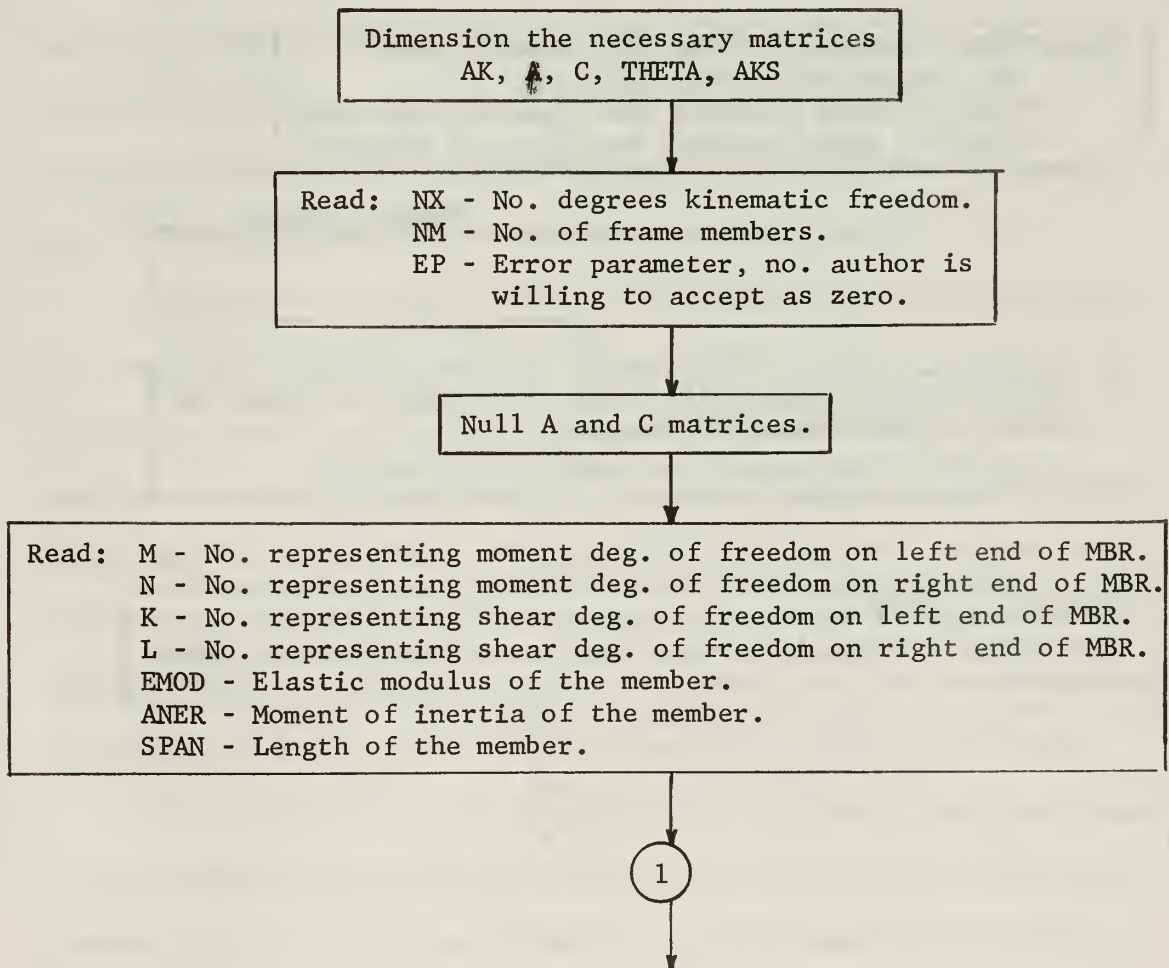
APPENDIX I

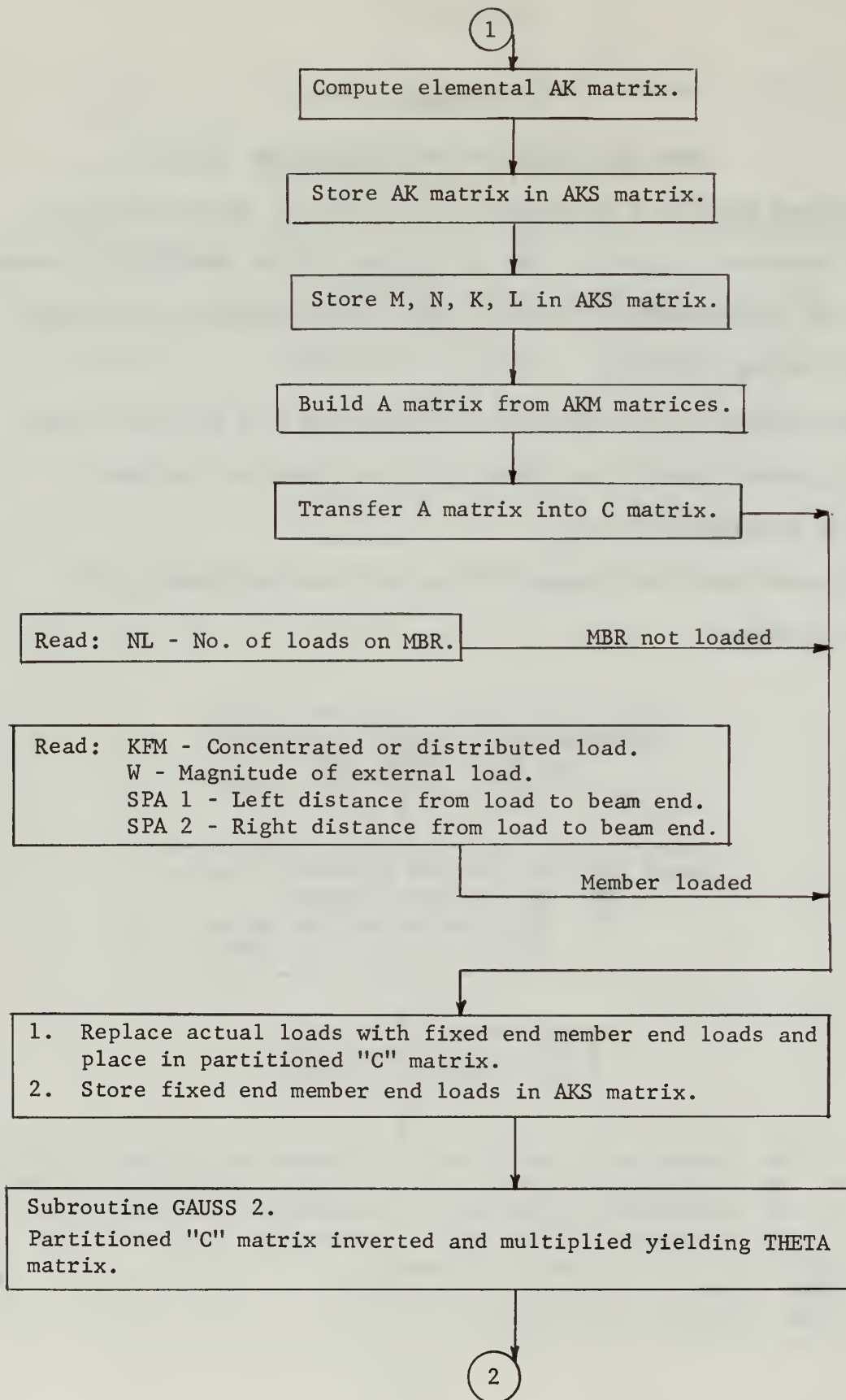
COMPUTER PROGRAM "FRAME" DESCRIPTION AND USE

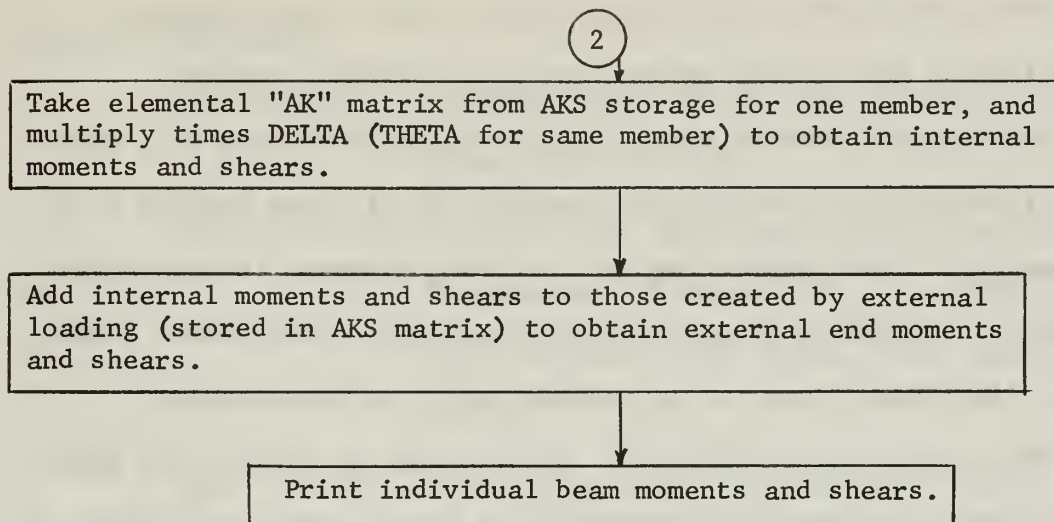
Program Frame is a self-contained, Fortran 60, digital computer program designed to provide joint deflections and/or rotation and member end forces and/or moments for plane rigid frame type structures under general loading conditions.

The geometry of the structure is restricted to a plane structure with all members parallel to either the X or Y axis in the usual cartesian coordinate system.

A generalized flow diagram showing the basic mechanics of the program follows.







The following information is for those who wish to actually use the program in its present form.

Properties such as the modules of elasticity (EMOD), the area moment of inertia (ANER), and the span may vary from one member to another; however, no provision is made for a single member with a variable moment of inertia (tapered member), except that an appropriate average might be used if the operator so desires.

Introduction of actual values of modulus of elasticity and moment of inertia will provide results, in appropriate units, of actual deflections and/or rotations. Or, if desired, the relative magnitude of modulus of elasticity and moment of inertia, based on selected references may be used instead of using the actual values. In this case, the results will indicate the rotation or deflection times a factor which is equal to $(E_{ref}) \times (I_{ref})$. In either case the correct values of member end forces and moments in appropriate units will result.

The required input data is relatively straight forward and simple in that the first data card describes the overall structure and subsequent cards describe the individual members taken one at a time.

Care should be taken in compiling the data cards, since seemingly insignificant errors will produce entirely erroneous results.

The program provides analysis for only one structure at a time. This structure may have only one complete set of loads applied to it. The number of loads comprising this one set, however, is essentially unlimited. Provision is made for application of concentrated loads and/or distributed loads to any member and in any combination.

The program output format is self-explanatory except that THETA may be actual rotations or deflections if actual values of modulus of elasticity and moment of inertia were used, or they may be multiples of the reference values if such reference values were used.

Following is the nomenclature of the program, a copy of the actual program in Fortran 60, a diagram for the model actually solved indicating member numbers and degrees of freedom (degrees of freedom indicated by arrows), data tabulation for the model as shown, a solution for deflections and rotations for the model, and the member end moments and shears for the model and data cards shown.

Nomenclature - (Program Proper)

AK - Stiffness matrix for an individual member.

A - Matrix used for obtaining the stiffness matrix for the complete structure.

C - Stiffness matrix for the complete structure.

THETA - Matrix of deflections and/or rotations.

AKS - Matrix used to store individual AK matrices and M, N, K, L, and end moments and forces due to external loads.

DELTA - THETA matrix for a single member.

F - Matrix of end forces and moments for a single member.

Required Input Data

- Data Card #1 Contains values of NX, NM, EP, punched in the format (2I10, E10.4).
- Data Card #2 Describes the first member to be considered. It contains values of M, N, K, L, EMOD, ANER, SPAN, punched in the format (4I2, 2E10.5, 1F10.5).
- Data Card #3 Defines the number of loads (NL) applied to the member described in card #2. NL is punched in the format (I2). If no load is applied to the member being considered, a zero must be punched in column number 2.
- Data Card #4 Contains values of KFM, W, SPA 1, SPA 2, punched in the format (I2, 3F10.5). If data card #3 set NL equal to zero, this card (#4) must be omitted. The total number of #4 cards used at this point must equal NL from card #3.

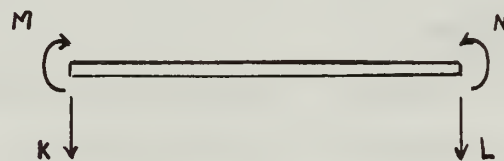
Subsequent Data Cards The input data card sequence is now repeated starting with card #2 above. Each member, in turn, must be completely described with its properties and loads before proceeding to the next member.

Nomenclature - (Input Data)

- NX - Number of degrees of kinematic deficiency.
- NM - Number of members or spans comprising the structure.
- EP - Error parameter, minimum acceptable value of the determinant of the stiffness matrix. If determinant is less than EP the program will stop and print "matrix singular."

- M - The number assigned to the rotational freedom at the left end of a member.
- N - The number assigned to the rotational freedom at the right end of a member.
- K - The number assigned to the translational freedom at the left end of a member.
- L - The number assigned to the translational freedom at the right end of a member.
- EMOD - Modulus of elasticity for a member. May also be used as a coefficient of some reference modulus.
- ANER - Area moment of inertia for a member. May also be used as a coefficient of some reference moment of inertia.
- SPAN - The length or span for a member.
- NL - The number of external loads (concentrated and/or uniform) applied to the single member being considered.
- KFM - Kind of load applied (1 = concentrated; 2 = uniform).
- W - Magnitude of the external load.
- SPA 1 - Distance from left end of member to the location of a concentrated load.
- SPA 2 - Distance from a concentrated load to the right end of a member.

Basic Element



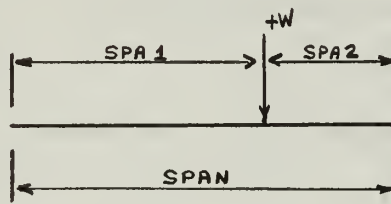
Note: Vertical members are rotated 90 degrees clockwise to determine the orientation indicated above.

The structure is sketched and all degrees of kinematic deficiency are numbered.

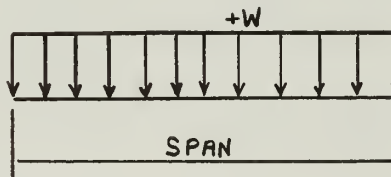
The individual members are viewed one at a time, as shown on previous page, and M, N, K, and L for each member are set equal to the number which was used to designate the corresponding degree of freedom for that member.

Load Parameters

$$KFM = 1$$



$$KFM = 2$$



```

PROGRAM FRAME
C RUN 110
  READ 11, NX, NM, EP
11 FORMAT (2I10, E10.4)
  DIMENSION AK(4,4), A(56,56), C(56,56), THETA(56), AKS(56,24),
1 DELTA(4), F(4)
  DO 90 IO = 1, NM
    READ 20, M, N, K, L, EMOD, ANER, SPAN
20 FORMAT (4I2, 2E10.5, 1F10.5)
    DO 23 I=1,56
      DO 23 J= 1,56
23 A(I,J)= 0.
      DO 26 I=1,4
        DO 26 J=1, 4
26 AK(I,J) = 0.
      AK(1,1)= (4.*EMOD*ANER)/SPAN
      AK(2,2)=AK(1,1)
      AK(1,2) = (2.*EMOD*ANER)/SPAN
      AK(1,3) = (6.*EMOD*ANER)/(SPAN**2)
      AK(2,3) = AK(1,3)
      AK(1,4) = -AK(1,3)
      AK(2,4) = -AK(1,3)
      AK(3,3) = (12.*EMOD*ANER)/(SPAN**3)
      AK(4,4) = AK(3,3)
      AK(3,4) = -AK(3,3)
      AK(2,1)=AK(1,2)
      AK(3,1)=AK(1,3)
      AK(3,2)=AK(2,3)
      AK(4,1)=AK(1,4)
      AK(4,2)=AK(2,4)
      AK(4,3)=AK(3,4)
      AKS(IO,17)=M
      AKS(IO,18)=N
      AKS(IO,19)=K
      AKS(IO,20)=L
      KK=1
      DO 3615 LL=1,4
        DO 3615 JJ=1,4
          AKS(IO, KK)=AK(LL, JJ)
3615 KK=KK+1
      IF (M) 103, 48, 38
38 A(M,M) = AK(1,1)
      IF (N) 103, 42, 40
40 A(M,N) = AK(1,2)
      A(N,M) = AK(1,2)
42 IF (K) 103, 45, 43
43 A(M,K) = AK(1,3)
      A(K,M) = AK(1,3)
45 IF (L) 103, 48, 46
46 A(M,L) = AK(1,4)
      A(L,M) = AK(1,4)
48 IF (N) 103, 56, 49
49 A(N,N) = AK(2,2)

```

```

      IF (K) 103,53, 51
51  A(N,K) = AK(2,3)
      A(K,N) = AK(2,3)
53  IF (L) 103, 56, 54
54  A(N,L) = AK(2,4)
      A(L,N) = AK(2,4)
56  IF (K) 103, 61, 57
57  A(K,K) = AK(3,3)
      IF (L) 103, 61, 59
59  A(K,L) = AK(3,4)
      A(L,K) = AK(3,4)
61  IF (L) 103, 63, 62
62  A(L,L) = AK(4,4)
63  DO 65 I=1,56
      DO 65 J=1,56
65  C(I,J) = C(I,J) + A(I, J)
      NPM=NX +1
      READ 681, NL
681  FORMAT (I2)
      IF (NL) 103, 90, 69
69  DO 90 KO = 1,NL
      READ 71, KFM, W, SPA1, SPA2
71  FORMAT (1I2, 3F10.5)
      IF (KFM-2) 73, 82, 103
73  IF (M)103,741,74
74  C(M,NPM) = (W*SPA1*SPA2**2)/(SPAN**2) + C(M,NPM)
741  AKS(IO,21)=-((W*SPA1*SPA2**2)/(SPAN**2) +AKS(IO,21)
      IF (N)103,761,76
76  C(N,NPM) = -(W*SPA2*SPA1**2)/(SPAN**2) + C(N,NPM)
761  AKS(IO,22)=(W*SPA2*SPA1**2)/(SPAN**2) +AKS(IO,22)
      IF (K) 103,781,78
78  C(K,NPM) = ((W*SPA2**2)*(3.*SPA1 + SPA2))/(SPAN**3) + C(K,NPM)
781  AKS(IO,23)=-((W*SPA2**2)*(3.*SPA1+SPA2))/(SPAN**3) +AKS(IO,23)
      IF (L) 103,801,80
80  C(L,NPM) = ((W*SPA1**2)*(3.*SPA2 + SPA1))/(SPAN**3) + C(L,NPM)
801  AKS(IO,24)=-((W*SPA1**2)*(3.*SPA2+SPA1))/(SPAN**3) +AKS(IO,24)
      GO TO 90
82  IF (M) 103,831,83
83  C(M,NPM) = (W*SPAN**2)/(12.) + C(M,NPM)
831  AKS(IO,21)=-((W*SPAN**2)/(12.) +AKS(IO,21)
      IF (N) 103,851,85
85  C(N,NPM) = -(W*SPAN**2)/(12.) + C(N,NPM)
851  AKS(IO,22)=(W*SPAN**2)/(12.) +AKS(IO,22)
      IF(K)103,871,87
87  C(K,NPM) = (W*SPAN)/(2.) + C(K,NPM)
871  AKS(IO,23)=-((W*SPAN)/(2.) +AKS(IO,23)
      IF(L) 103,891,89
89  C(L,NPM) = (W*SPAN)/(2.) + C(L,NPM)
891  AKS(IO,24)=-((W*SPAN)/(2.) +AKS(IO,24)
90  CONTINUE
      NP = 1
      CALL GAUSS2 (NX, NP, EP, E, THETA, K1)
      GO TO (93, 99) , K1

```



```

93 PRINT 94
94 FORMAT(////44H1NOTE 1. THETA IS JOINT ROTATION/DEFLECTION/)
PRINT 321
321 FORMAT (29H NOTE 2. ANSWER = EI X THETA////)
DO 96 K=1,NX
96 PRINT 97, K, THETA(K)
97 FORMAT (11H0EI X THETA 13.3H = F20.15)
PRINT 9072
9072 FORMAT(////43H1MEMBER END MOMENTS AND SHEARS APPEAR BELOW/)
PRINT 9074
9074 FORMAT(47H CLOCKWISE MOMENTS AND DOWN SHEARS ARE POSITIVE////)
PRINT 730
730 FORMAT(////43H          LEFT      RIGHT      LEFT      RIGHT)
PRINT 731
731 FORMAT(43H MEMBER      MOMENT      MOMENT      SHEAR      SHEAR//)
DO 9735 IM=1,NM
KJ= 1
DO 976 LL=1,4
DO 976 JJ=1,4
AK(LL,JJ)=AKS(IM,KJ)
976 KJ=KJ +1
M=AKS(IM,17)
IF (M) 103,9711,979
979 DELTA(1)=THETA(M)
GO TO 9712
9711 DELTA(1)=0.
9712 N=AKS(IM,18)
IF(N) 103,9716,9714
9714 DELTA(2)= THETA(N)
GO TO 9717
9716 DELTA(2)=0.
9717 K=AKS(IM,19)
IF (K) 103,9721,9719
9719 DELTA(3) =THETA(K)
GO TO 9722
9721 DELTA(3)=0.
9722 L=AKS(IM,20)
IF (L)103,9726,9724
9724 DELTA(4)=THETA(L)
GO TO 9727
9726 DELTA(4)=0.
9727 DO 9730 KB=1,4
F(KB)=0.
DO 9730 JB=1,4
9730 F(KB)=F(KB) +AK(KB,JB) * DELTA(JB)
F(1)=F(1) +AKS(IM,21)
F(2)=F(2) + AKS(IM,22)
F(3)=F(3) + AKS(IM,23)
F(4)=F(4) + AKS(IM,24)
9735 PRINT 9736, IM,F(1),F(2),F(3),F(4)
9736 FORMAT (14, F9.3,3F7.3)
GO TO 105
99 PRINT 100

```



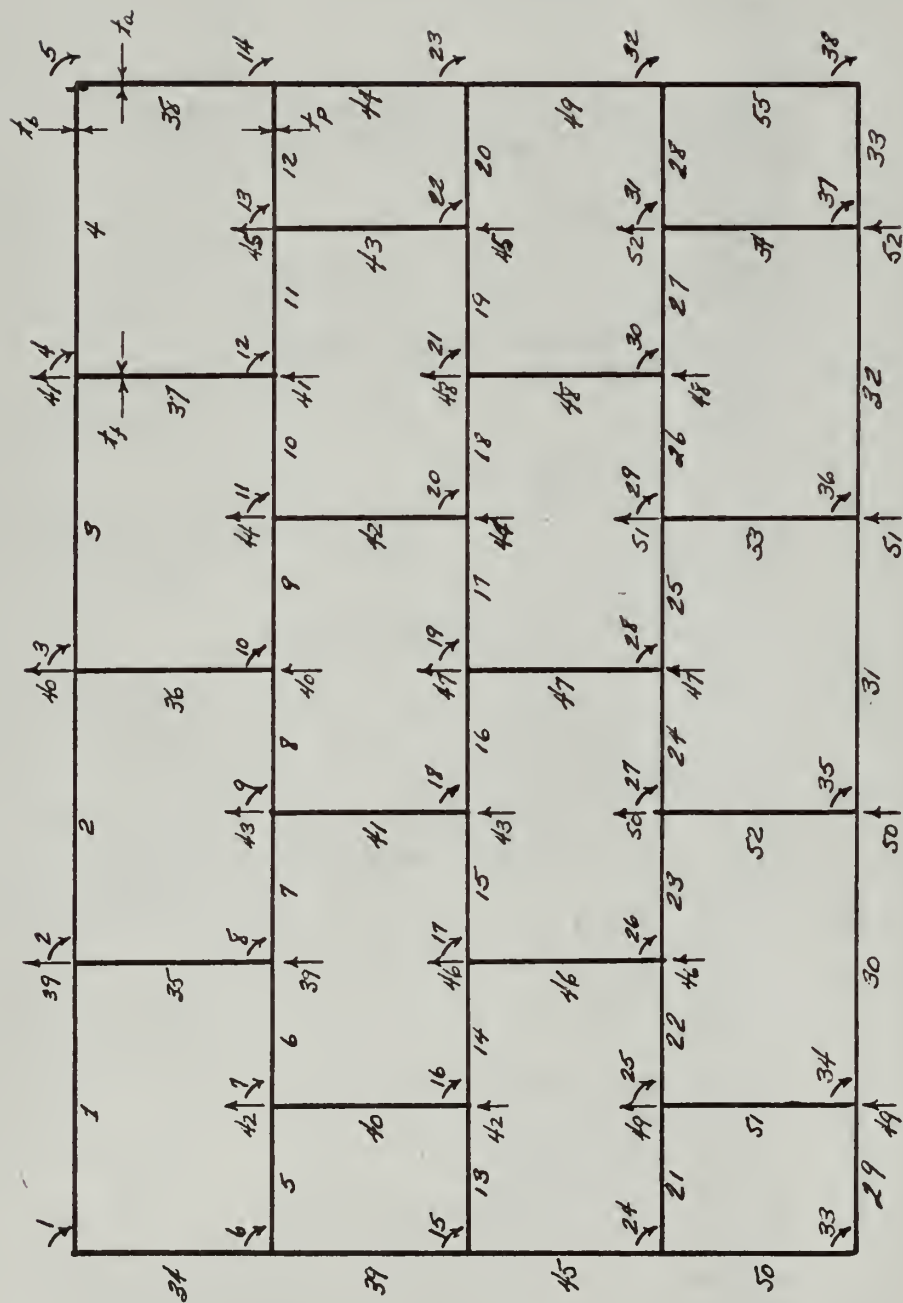
```

100 FORMAT (16H MATRIX SINGULAR)
    GO TO 105
103 PRINT 104
104 FORMAT (12H INPUT ERROR)
105 CONTINUE
    STOP
    END
    SUBROUTINE GAUSS2(N,M,EP,A,X,KER)
    DIMENSION A(56,56),X(56)
    NPM=N+M
10 DO 34 L=1,N
    KP=0
    Z=0.0
    DO 12 K=L,N
    IF(Z-ABSF(A(K,L)))11,12,12
11 Z=ABSF(A(K,L))
    KP=K
12 CONTINUE
    IF(L-KP)13,20,20
13 DO 14 J=L,NPM
    Z=A(L,J)
    A(L,J)=A(KP,J)
14 A(KP,J)=Z
20 IF(ABSF(A(L,L))-EP)50,50,30
30 IF(L-N)31,40,40
31 LP1=L+1
    DO 34 K=LP1,N
    IF(A(K,L))32,34,32
32 RATIO=A(K,L)/A(L,L)
    DO 33 J=LP1,NPM
33 A(K,J)=A(K,J)-RATIO*A(L,J)
34 CONTINUE
40 DO 43 I=1,N
    II=N+1-I
    DO 43 J=1,M
    JPN=J+N
    S=0.0
    IF(II-N)41,43,43
41 IIP1=II+1
    DO 42 K=IIP1,N
42 S=S+A(II,K)*X(K)
43 X(II)=(A(II,JPN)-S)/A(II,II)
    KER=1
    RETURN
50 KER = 2
    END
    END

```

H

•



| | 52 | 55 | .1E-07 | |
|----------|----------|----|--------|------|
| 1020039 | 51.2E 01 | | .1E 01 | .75 |
| 1 | | | | |
| 2 | -150. | 0. | 0. | |
| 2033940 | 51.2E 01 | | .1E 01 | .75 |
| 1 | | | | |
| 2 | -150. | 0. | 0. | |
| 3044041 | 51.2E 01 | | .1E 01 | .75 |
| 1 | | | | |
| 2 | -150. | 0. | 0. | |
| 4054100 | 51.2E 01 | | .1E 01 | .75 |
| 1 | | | | |
| 2 | -150. | 0. | 0. | |
| 6070042 | .8E 01 | | .1E 01 | .375 |
| 7084239 | .8E 01 | | .1E 01 | .375 |
| 8093943 | .8E 01 | | .1E 01 | .375 |
| 9104340 | .8E 01 | | .1E 01 | .375 |
| 10114044 | .8E 01 | | .1E 01 | .375 |
| 11124441 | .8E 01 | | .1E 01 | .375 |
| 12134145 | .8E 01 | | .1E 01 | .375 |
| 13144500 | .8E 01 | | .1E 01 | .375 |
| 15160042 | .8E 01 | | .1E 01 | .375 |
| 16174246 | .8E 01 | | .1E 01 | .375 |
| 17184643 | .8E 01 | | .1E 01 | .375 |
| 18194347 | .8E 01 | | .1E 01 | .375 |
| 19204744 | .8E 01 | | .1E 01 | .375 |
| 20214448 | .8E 01 | | .1E 01 | .375 |
| 21224845 | .8E 01 | | .1E 01 | .375 |
| 22234500 | .8E 01 | | .1E 01 | .375 |
| 24250049 | .8E 01 | | .1E 01 | .375 |
| 25264946 | .8E 01 | | .1E 01 | .375 |
| 26274650 | .8E 01 | | .1E 01 | .375 |

| | | | |
|----------|-----------|--------|------|
| 27285047 | .8E 01 | .1E 01 | .375 |
| 28294751 | .8E 01 | .1E 01 | .375 |
| 29305148 | .8E 01 | .1E 01 | .375 |
| 30314852 | .8E 01 | .1E 01 | .375 |
| 31325200 | .8E 01 | .1E 01 | .375 |
| 33340049 | 51.2E 01 | .1E 01 | .375 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 34354950 | 51.2E 01 | .1E 01 | .75 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 35365051 | 51.2E 01 | .1E 01 | .75 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 36375152 | 51.2E 01 | .1E 01 | .75 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 37385200 | 51.2E 01 | .1E 01 | .375 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 6010000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | -150. | 0. | 0. |
| 8020000 | .0125E 01 | .1E 01 | .5 |
| 10030000 | .0125E 01 | .1E 01 | .5 |
| 12040000 | .0125E 01 | .1E 01 | .5 |
| 14050000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 15060000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | -150. | 0. | 0. |
| 16070000 | .0125E 01 | .1E 01 | .5 |
| 18090000 | .0125E 01 | .1E 01 | .5 |
| 20110000 | .0125E 01 | .1E 01 | .5 |
| 22130000 | .0125E 01 | .1E 01 | .5 |

| | | | |
|----------|-----------|--------|----|
| 23140000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 24150000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | -150. | 0. | 0. |
| 26170000 | .0125E 01 | .1E 01 | .5 |
| 28190000 | .0125E 01 | .1E 01 | .5 |
| 30210000 | .0125E 01 | .1E 01 | .5 |
| 32230000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | 150. | 0. | 0. |
| 33240000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | -150. | 0. | 0. |
| 34250000 | .0125E 01 | .1E 01 | .5 |
| 35270000 | .0125E 01 | .1E 01 | .5 |
| 36290000 | .0125E 01 | .1E 01 | .5 |
| 37310000 | .0125E 01 | .1E 01 | .5 |
| 38320000 | .8E 01 | .1E 01 | .5 |
| 1 | | | |
| 2 | 150. | 0. | 0. |

NOTE 1. THETA IS JOINT ROTATION/DEFLECTION

NOTE 2. ANSWER = EI X THETA

| | | |
|------------|------|--------------------|
| EI X THETA | 1 = | -.059757168141914 |
| EI X THETA | 2 = | -.039902351722048 |
| EI X THETA | 3 = | -.0000000000003752 |
| EI X THETA | 4 = | .039902351729324 |
| EI X THETA | 5 = | .059757168116448 |
| EI X THETA | 6 = | .020913817393648 |
| EI X THETA | 7 = | -.084237065282650 |
| EI X THETA | 8 = | -.000181155194702 |
| EI X THETA | 9 = | -.030079360703894 |
| EI X THETA | 10 = | .0000000000006821 |
| EI X THETA | 11 = | .030079360741638 |
| EI X THETA | 12 = | .000181155228802 |
| EI X THETA | 13 = | .084237065302659 |
| EI X THETA | 14 = | -.020913817406381 |
| EI X THETA | 15 = | -.023057530810092 |
| EI X THETA | 16 = | .038969665726654 |
| EI X THETA | 17 = | -.041627451475506 |
| EI X THETA | 18 = | .012380074253315 |
| EI X THETA | 19 = | -.0000000000023057 |
| EI X THETA | 20 = | -.012380074174189 |
| EI X THETA | 21 = | .041627451528257 |
| EI X THETA | 22 = | -.038969665821242 |
| EI X THETA | 23 = | .023057530988808 |
| EI X THETA | 24 = | .025138163619431 |
| EI X THETA | 25 = | .003051132390510 |
| EI X THETA | 26 = | .053146538107285 |
| EI X THETA | 27 = | -.011316232965555 |
| EI X THETA | 28 = | -.0000000000938599 |
| EI X THETA | 29 = | .011316233752268 |
| EI X THETA | 30 = | -.053146539722547 |
| EI X THETA | 31 = | -.003051131717825 |
| EI X THETA | 32 = | -.025138164648979 |
| EI X THETA | 33 = | .049425681714638 |
| EI X THETA | 34 = | .047713020395349 |
| EI X THETA | 35 = | .018272389301728 |
| EI X THETA | 36 = | -.018272389324466 |
| EI X THETA | 37 = | -.047713020370793 |
| EI X THETA | 38 = | -.049425681679168 |
| EI X THETA | 39 = | -.039716653672258 |
| EI X THETA | 40 = | -.054848966682584 |
| EI X THETA | 41 = | -.039716653668620 |
| EI X THETA | 42 = | -.005222375323569 |
| EI X THETA | 43 = | -.019661487614485 |
| EI X THETA | 44 = | -.019661487617668 |
| EI X THETA | 45 = | -.005222375324706 |
| EI X THETA | 46 = | .011389984892958 |
| EI X THETA | 47 = | .012368798751140 |
| EI X THETA | 48 = | .011389984899324 |
| EI X THETA | 49 = | .018564758140656 |
| EI X THETA | 50 = | .044199614962963 |
| EI X THETA | 51 = | .044199614952049 |
| EI X THETA | 52 = | .018564758128377 |

MEMBER END MOMENTS AND SHEARS APPEAR BELOW
CLOCKWISE MOMENTS AND DOWN SHEARS ARE POSITIVE

| MEMBER | LEFT MOMENT | RIGHT MOMENT | LEFT SHEAR | RIGHT SHEAR |
|--------|----------------|-----------------|---------------|----------------|
| 1 | 6.28 | 19.33 | 90.39 | 22.11 |
| 2 | -19.29 | 21.13 | 58.71 | 53.79 |
| 3 | -21.13 | 19.29 | 53.79 | 58.71 |
| 4 | -19.33 | -6.28 | -22.11 | 90.39 |
| 5 | -.03 | -4.51 | -12.11 | 12.11 |
| 6 | 4.58 | 8.16 | -33.98 | -33.98 |
| 7 | -8.14 | -9.42 | -46.84 | 46.84 |
| 8 | 9.44 | 10.73 | 53.79 | -53.79 |
| 9 | -10.73 | -9.44 | -53.79 | 53.79 |
| 10 | 9.42 | 8.14 | 46.84 | -46.84 |
| 11 | -8.16 | -4.58 | -33.98 | 33.98 |
| 12 | 4.51 | .03 | 12.11 | -12.11 |
| 13 | 1.48 | 4.12 | 14.94 | -14.94 |
| 14 | -4.12 | -7.56 | -31.15 | 31.15 |
| 15 | 7.57 | 9.88 | 46.54 | -46.54 |
| 16 | -9.88 | -10.40 | -54.08 | 54.08 |
| 17 | 10.40 | 9.88 | 54.08 | -54.08 |
| 18 | -9.88 | -7.57 | -46.54 | 46.54 |
| 19 | 7.56 | 4.12 | 31.15 | -31.15 |
| 20 | -4.12 | -1.48 | -14.94 | 14.94 |
| 21 | -4.06 | -5.00 | -24.17 | 24.17 |
| 22 | 4.98 | 7.11 | 32.24 | -32.24 |
| 23 | -7.15 | -9.90 | -45.45 | 45.45 |
| 24 | 9.90 | 10.38 | 54.08 | -54.08 |
| 25 | -10.38 | -9.90 | -54.08 | 54.08 |
| 26 | 9.90 | 7.15 | 45.45 | -45.45 |
| 27 | -7.11 | -4.98 | -32.24 | 32.24 |
| 28 | -5.00 | -4.06 | -24.17 | 24.17 |
| 29 | -7.09 | -8.25 | -69.05 | 12.80 |
| 30 | 8.20 | -17.93 | -69.22 | -43.28 |
| 31 | 17.92 | -17.92 | -56.25 | -56.25 |
| 32 | 17.93 | -8.20 | -43.28 | -69.22 |
| 33 | 8.25 | 7.09 | 12.80 | -69.05 |
| 34 | 2.55 | -6.28 | 30.04 | 44.96 |
| 35 | -.02 | -.04 | -.12 | -.12 |
| 36 | .00 | -.00 | .00 | -.00 |
| 37 | .02 | .04 | .12 | -.12 |
| 38 | -2.55 | 6.28 | -30.04 | -44.96 |
| 39 | 2.32 | -2.52 | 37.09 | 37.91 |
| 40 | -.00 | -.06 | -.14 | .14 |
| 41 | -.00 | -.02 | -.05 | .05 |
| 42 | .00 | .02 | .05 | -.05 |
| 43 | .00 | .06 | .14 | -.14 |
| 44 | -2.32 | 2.52 | -37.09 | -37.91 |
| 45 | 4.00 | -3.80 | 37.90 | 37.10 |
| 46 | .03 | -.02 | .03 | -.03 |
| 47 | -.00 | -.00 | -.00 | .00 |
| 48 | -.03 | .02 | .03 | .03 |
| 49 | -4.00 | 3.80 | -37.90 | -37.10 |
| 50 | 7.09 | .07 | 51.82 | 23.18 |
| 51 | .05 | .03 | .15 | -.15 |
| 52 | .01 | -.00 | .02 | -.02 |
| 53 | -.01 | .00 | -.02 | .02 |
| 54 | -.05 | -.03 | -.15 | .15 |
| 55 | -7.09 | -.07 | -51.82 | -23.18 |

STOP
TIME, 1 MINUTES AND 40 SECONDS

APPENDIX II

OTHER METHODS OF ANALYSIS

Prior to the decision to use the two-dimensional frame analysis for the internal pressure stresses a number of other methods were investigated. Generally, the approach was one of searching for the correct design philosophy and attempting to adapt simple solutions to the problem at hand.

The methods outlined herein seemed to apply to the problem but in each case the results became too complex to provide useful design information or the results required specific knowledge of some particular parameter which was unavailable.

TOP PLATE ON AN ELASTIC FOUNDATION

The first approach was to consider the top plate of the heat exchanger to be fixed on the two sides adjoining the side closure bars, to be resting on an elastic foundation, with modulus K lb/in, along its span, and to be loaded with a uniform pressure load, Q lb/in. The elastic foundation was assumed to be the result of the arrangement of interior plates and fins. Assuming no change in geometry or other parameters with length, the plate was analyzed as a beam considering the stiffness to be $EI/(1-\nu^2)$ instead of simply EI .

The expression for the moment at the end of this beam is:

$$M = \frac{P}{2\lambda^2} \left[\frac{\sinh \lambda b - \sin \lambda b}{\sinh \lambda b + \sin \lambda b} \right]$$

where:

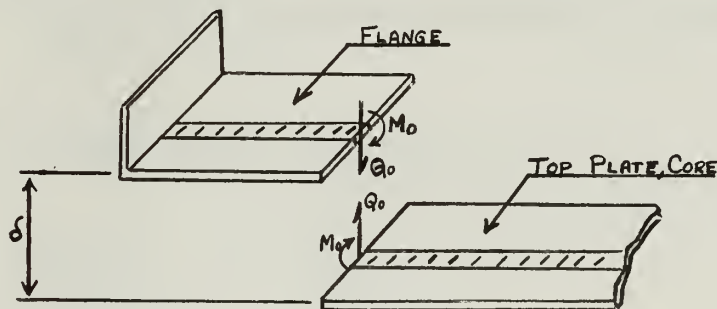
$$\lambda = \sqrt[4]{\frac{K(1-\nu^2)}{4EI}}$$

This result shows that the modulus of the elastic foundation, K , is of primary importance. In truth, this modulus is a function of such parameters as fin spacing, width of the heat exchanger, and thickness or

stiffness of the interior plates. No adequate analytical means of estimating the magnitude of the foundation modulus was found. It is recommended that K be determined by testing actual modules.

DISCONTINUITIES BETWEEN HEADER AND CORE

A number of attempts were made to determine the value of discontinuity stresses at the joint between the header or flange and core. In general, this method attempted to extend the classical solutions for cylindrical pressure vessels to include rectangular pressure vessels. This approach might be possible if the corners of the rectangular heat exchanger could be rounded off slightly such that a continuous mathematical function could be used to define the surface of the heat exchanger. The best approach with the square corners appears to be with the use of an elemental lengthwise strip taken at the location of the greatest discontinuity in deformation between the core and the flange as shown below.



If Q_0 and M_0 could be determined, the elemental strip of the top plate of the core could be analyzed as a beam on an elastic foundation with end loads Q_0 and M_0 . Difficulties, in connection with finding Q_0 and M_0 , the magnitude of the elastic foundation modulus, whether the elemental strip of the core could be considered semi-infinite in length, and the correct method for determining the δ between the header and core, were encountered.

FRAME ANALYSIS

A number of lattice-type frames were analyzed and the analysis for the simple frame with two axis of symmetry did, in fact, provide the impetus for continuing with the complex frame analysis as used. This analysis provided fundamental information as to the effect of the various parameters involved in the core.

Frames with various numbers of non-yielding tie bars, which represented the interior plates, were analyzed. The solutions neglected the effect of fins. This approach was abandoned because of the inability to adequately account for the effect of the fins.

SANDWICH CONSTRUCTION

Considerable effort was expended in researching publications dealing with sandwich construction. Much is available on the subject in the literature but no reference was found which dealt with sandwich or honeycomb structures loaded with an internal pressure.

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| <p>Regenerators have been proposed to increase thermal cycle efficiency of gas turbine power plants which are being considered for installation in certain classes of United States Navy ships. These regenerators are to preheat intake air and are currently designed as compact heat exchangers. The recommended heat exchanger configurations are either counterflow or crossflow. This analysis considers only the counterflow configuration.</p> <p>Some of the structural problems associated with rectangular, compact heat exchanger design are discussed and certain structural design parameters are determined analytically. A digital computer program is presented for the analysis of the pressure effects in an idealized, two dimensional, rectangular heat exchanger. This program is written for a matrix analysis of an idealized heat exchanger with a cross section analogous to a multi-story building frame.</p> <p>This investigation was conducted by Robert L. Corbett and David W. Stubbs during the period from December 1965 to May 1966, at the United States Naval Postgraduate School, Monterey, California.</p> | | | |

| 14. KEY WORDS | LINK A | | LINK B | | LINK C | |
|----------------|--------|----|--------|----|--------|----|
| | ROLE | WT | ROLE | WT | ROLE | WT |
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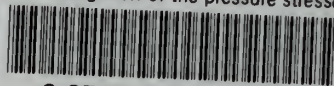
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